

2. cvičení - výsledky

Příklad 1.

- | | | |
|----------------|----------------|--------------|
| (a) je komp. | (g) je komp. | (m) je komp. |
| (b) je komp. | (h) není komp. | (n) je komp. |
| (c) není komp. | (i) není komp. | (o) je komp. |
| (d) je komp. | (j) není komp. | (p) je komp. |
| (e) je komp. | (k) je komp. | (q) je komp. |
| (f) je komp. | (l) je komp. | |

Příklad 2.

- (a) $\frac{x^3+x^2+4x}{2(x-1)(x+1)^2\sqrt{x^3-1}}$
- (b) $-\tan(e^x) \cdot e^x$
- (c) $6 \cdot (\cos(2x) - e^{2x})^{-4} \cdot (\sin(2x) + e^{2x})$
- (d) $\frac{\log(x^3-2x+1)}{x\sqrt{x^2-1}} + \arccos\left(\frac{1}{x}\right) \cdot \frac{3x^2-2}{(x^3-2x+1)}$

Příklad 3.

- (a) $f'_x(x, y) = 6xy + 35, [x, y] \in \mathbb{R}^2$
 $f'_y(x, y) = 8y + 3x^2, [x, y] \in \mathbb{R}^2$
- (b) $f'_x(x, y) = -\frac{\sin y^2}{x^2}, \{[x, y] \in \mathbb{R}^2 : x \neq 0\}$
 $f'_y(x, y) = \frac{2y \cos y^2}{x}, \{[x, y] \in \mathbb{R}^2 : x \neq 0\}$
- (c) $f'_x(x, y) = ye^{xy} + \cos(x+y), [x, y] \in \mathbb{R}^2$
 $f'_y(x, y) = xe^{xy} + \cos(x+y), [x, y] \in \mathbb{R}^2$
- (d) $f'_x(x, y) = \tan \frac{x}{y} + \frac{x}{y \cos^2 \frac{x}{y}}, \{[x, y] \in \mathbb{R}^2 : y \neq 0, \frac{x}{y} \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\}$
 $f'_y(x, y) = \frac{-x^2}{y^2 \cos^2 \frac{x}{y}}, \{[x, y] \in \mathbb{R}^2 : y \neq 0, \frac{x}{y} \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\}$
- (e) $f'_x(x, y) = yx^{y-1}, \{[x, y] \in \mathbb{R}^2 : x > 0\}$
 $f'_y(x, y) = x^y \log x, \{[x, y] \in \mathbb{R}^2 : x > 0\}$
- (f) $f'_x(x, y) = \frac{x^2}{\sqrt[3]{(x^3+y^3)^2}}, \{[x, y] \in \mathbb{R}^2 : y \neq -x\}$
 $f'_y(x, y) = \frac{y^2}{\sqrt[3]{(x^3+y^3)^2}}, \{[x, y] \in \mathbb{R}^2 : y \neq -x\}$

$$(g) \quad f'_x(x, y) = |y| \cdot \text{sign } x, \{[x, y] \in \mathbb{R}^2 : x \neq 0\}$$

$$f'_y(x, y) = |x| \cdot \text{sign } y, \{[x, y] \in \mathbb{R}^2 : y \neq 0\}$$

$$f'_x(0, 0) = f'_y(0, 0) = 1$$

Ve zbylých bodech derivace neexistují.

Dále $f'_x(0, 0) = f'_y(0, 0) = 0$ a zbylé derivace neexistují.

$$(h) \quad f'_x(x, y) = \frac{2x}{3\sqrt[3]{(x^2+y^2)^2}} \log(x^2 + y^2) + 2x \frac{\sqrt[3]{x^2+y}}{x^2+y^2}, \{[x, y] \in \mathbb{R}^2 : y \neq -x^2\}$$

$$f'_y(x, y) = \frac{1}{3\sqrt[3]{(x^2+y^2)^2}} \log(x^2 + y^2) + 2y \frac{\sqrt[3]{x^2+y}}{x^2+y^2}, \{[x, y] \in \mathbb{R}^2 : y \neq -x^2\}$$

Dále pokud $x^2 + x^4 \neq 1$, pak paricální derivace neexistují v bodě $(x, -x^2)$.

Pokud $x^2 + x^4 = 1$, pak $f'_x(x, -x^2) = f'_y(x, -x^2) = 0$

$$(i) \quad f'_x(x, y) = e^{\frac{-1}{x^2+xy+y^2}} \cdot \frac{2x+y}{(x^2+xy+y^2)^2}, [x, y] \in \mathbb{R}^2 \setminus \{[0, 0]\}$$

$$f'_y(x, y) = e^{\frac{-1}{x^2+xy+y^2}} \cdot \frac{x+2y}{(x^2+xy+y^2)^2}, [x, y] \in \mathbb{R}^2 \setminus \{[0, 0]\}$$

$$A \quad f'_x(0, 0) = f'_y(0, 0) = 0$$

Příklad 4.

$$(a) \quad f'_x(x, y, z) = yz + \log z, \{[x, y] \in \mathbb{R}^2 : z > 0\}$$

$$f'_y(x, y, z) = xz + z^2, \{[x, y] \in \mathbb{R}^2 : z > 0\}$$

$$f'_z(x, y, z) = xy + \frac{x}{z} + 2yz, \{[x, y] \in \mathbb{R}^2 : z > 0\}$$

$$(b) \quad f'_x(x, y) = -\frac{y}{x^2-y^2}, \{[x, y] \in \mathbb{R}^2 : -x < y < x \vee x < y < -x\}$$

$$f'_y(x, y) = \frac{x}{x^2-y^2}, \{[x, y] \in \mathbb{R}^2 : -x < y < x \vee x < y < -x\}$$

$$(c) \quad f'_x(x, y) = \frac{y}{x^2+y^2}, \{[x, y] \in \mathbb{R}^2 : x \neq y\}$$

$$f'_y(x, y) = -\frac{x}{x^2+y^2}, \{[x, y] \in \mathbb{R}^2 : x \neq y\}$$

$$(d) \quad f'_x(x, y) = y \cdot \text{sign } x, \{[x, y] \in \mathbb{R}^2 : x \neq 0\}, \text{ dále } f'_x(0, 0) = 0 \text{ a } f'_x \text{ neexistuje pro } x = 0, y \neq 0$$

$$f'_y(x, y) = |x|, [x, y] \in \mathbb{R}^2$$

$$(e) \quad f'_x(x, y) = \frac{\sqrt[3]{y}}{3\sqrt[3]{x^2}}, \{[x, y] \in \mathbb{R}^2 : x \neq 0\}$$

$$f'_y(x, y) = \frac{\sqrt[3]{x}}{3\sqrt[3]{y^2}}, \{[x, y] \in \mathbb{R}^2 : y \neq 0\}$$

Dále $f'_x(0, 0) = f'_y(0, 0) = 0$ a zbylé derivace neexistují.

$$(f) \quad f'_x(x, y) = -\text{sign}(y - \sin x) \cdot \cos x, \{[x, y] \in \mathbb{R}^2 : y \neq \sin x\} \text{ a } f'_x\left(\frac{\pi}{2} + k\pi, (-1)^k\right) = 0, k \in \mathbb{Z}$$

$$f'_y(x, y) = \text{sign}(y - \sin x), \{[x, y] \in \mathbb{R}^2 : y \neq \sin x\}$$

$$\begin{aligned}
\text{(g)} \quad f'_x(x, y, z) &= \frac{z}{y} \left(\frac{x}{y} \right)^{z-1}, \{[x, y] \in \mathbb{R}^3 : xy > 0\} \\
f'_y(x, y, z) &= -\frac{xz}{y^2} \left(\frac{x}{y} \right)^{z-1}, \{[x, y] \in \mathbb{R}^3 : xy > 0\} \\
f'_z(x, y, z) &= \left(\frac{x}{y} \right)^z \log \frac{x}{y}, \{[x, y] \in \mathbb{R}^3 : xy > 0\} \\
\text{(h)} \quad f'_x(x, y, z) &= \frac{y}{z} \cdot x^{\frac{y}{z}-1}, \{[x, y] \in \mathbb{R}^3 : x > 0, z \neq 0\} \\
f'_y(x, y, z) &= x^{\frac{y}{z}} \cdot \frac{\log x}{z}, \{[x, y] \in \mathbb{R}^3 : x > 0, z \neq 0\} \\
f'_z(x, y, z) &= -x^{\frac{y}{z}} \cdot \frac{y \log x}{z^2}, \{[x, y] \in \mathbb{R}^3 : x > 0, z \neq 0\}
\end{aligned}$$